Fig. 1.

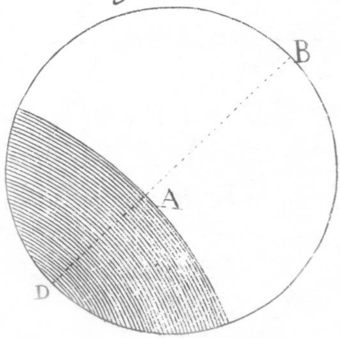


Fig. 2.

Fig. 3.

Fig. 5.

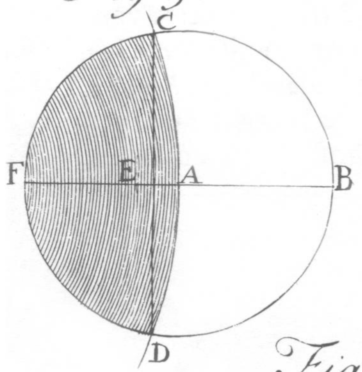


Fig. 4.

Fig. 6.

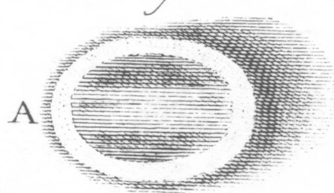
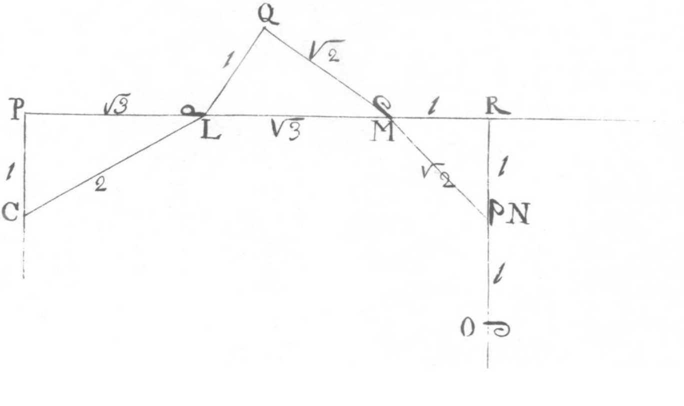


Fig. 7.



I. *A short Account of Negativo-affirmative Arithmetick, by Mr.* John Colson, F. R. S.

THE Usefulness of this Arithmetick consists in this, that it performs all the Operations with more Ease and Expedition than the common Affirmative Arithmetick, especially in large Numbers: And it differs from the common Arithmetick chiefly in this, that it admits of Negative Figures promiscuously with the affirmative. These negative Figures are distinguish'd from the Affirmative, by the Sign $-$ placed over them. Thus $3\bar{7}09\bar{2}\bar{8}65\bar{7}3\bar{9}\bar{6}\bar{1}47\bar{2}$ is one of these Numbers, which may be converted into its Equivalent common Number 2308726432039468, in this manner:

$$
\begin{array}{r}
3009006503000470 \\
0700280070961002 \\
\hline
2308726432039468
\end{array}
$$

(1.) Write down all the affirmative Figures by themselves, putting a Cypher in the place of every negative Figure. (2.) Write down all the negative Figures by themselves, putting a Cypher in the place of every affirmative Figure. (3.) Subtract the last Number from the first, and the Remainder will be a common Number, equivalent to the given negativo-affirmative Number. See the Operation above.

But the readiest practical Way of performing this Reduction in any given Number, will be in this manner: Begin at the left hand; and going over all the Figures in order, observe these Rules. (1.) An affirmative

firmative Figure before a Negative muſt be diminiſh'd by an Unit. (2.) A negative Figure before an Affirmative, muſt be changed into its Complement to 10. (3.) A negative Figure before a Negative, muſt be changed into its Complement to 9. All other Figures muſt remain unchanged, and a Cypher is always to be underſtood where there is no ſignificant Figure. The ſign of the Cypher is neglected ; but where there is occaſion to conſider it, it is always ſuppos'd the ſame as the Sign of the following Figure. Thus the negativo-affirmative Number $72\bar{9}\bar{5}86\bar{4}\bar{5}\bar{5}98\bar{2}00\bar{1}\bar{7}30$ is immediately reduc'd to 710585545977998330; and ſo of all others.

But on the contrary, common Numbers may be reduced to negativo-affirmative Numbers a great variety of Ways, by ſubſtituting inſtead of the Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, their reſpective Values $1\bar{9}$, $1\bar{8}$, $1\bar{7}$, $1\bar{6}$, $1\bar{5}$, $1\bar{4}$, $1\bar{3}$, $1\bar{2}$, $1\bar{1}$, in any places at pleaſure. But the moſt uſeful Reduction of this kind is what I call *A Reduction to ſmall Figures*, which conſiſts in throwing out all the large Figures, 9, 8, 7, 6, out of any given Number, and introducing in their room the equivalent ſmall Figures $1\bar{1}$, $1\bar{2}$, $1\bar{3}$, $1\bar{4}$, reſpectively. Thus 182937462 may be reduc'd to $2\bar{2}3\bar{1}4\bar{3}5\bar{4}2$, conſiſting only of ſmall Figures. But this Reduction may be perform'd more readily by theſe Rules following.

(1.) A ſmall Figure before a large Figure muſt be increaſed by an Unit. (2.) A large Figure before a large Figure muſt be changed into its negative Complement to 9. (3.) A large Figure before a ſmall Figure muſt be changed into its negative Complement to 10. Other Figures are not to be changed ; and 5 will be ambiguous, being to be eſteemed either large

or

or ſmall, according as the Figure following is either large or ſmall. Some Examples of this Reduction ſhall here follow, both in whole Numbers and Decimal Fractions.

$$
\begin{array}{rcl}
37068259764 & = & 4\bar{3}1\bar{3}\bar{2}3\bar{4}0\bar{2}\bar{4}4 \\
729528960739957 & = & 1\bar{3}3\bar{1}53\bar{1}0\bar{4}1\bar{3}400\bar{4}\bar{3} \\
9260872395{,}87294 & = & 1\bar{1}3\bar{4}1\bar{1}\bar{3}240\bar{4}{,}\bar{1}\bar{3}3\bar{1}4 \\
\text{Or}\,(9)\,926087239587294 & = & (10)\ 1\bar{1}3\bar{4}1\bar{1}\bar{3}240\bar{4}\bar{1}\bar{3}3\bar{1}4 \\
(m)\,387916407953,\&c. & = & (m)\ 4\bar{1}\bar{2}\bar{1}2\bar{4}41\bar{2}\bar{1}53,\ \&c.
\end{array}
$$

It is to be obſerved, that in this laſt Example the Numbers are what I call *interminate*, or Approximations only; that is, the firſt and moſt valuable Figures are expreſs'd, and all the reſt (whether finite or infinite in Number, whether known or unknown) are omitted as inconſiderable, and inſinuated by the Mark &c. Alſo the Index *m* before the Number ſtands for ſome Integer, expreſſing the Diſtance of the firſt Figure 3 or 4 from the Place of Units; which Integer is either affirmative or negative, according as the ſaid firſt Figure ſtands in integral or fractional Places. The Example immediately before is a particular Inſtance of this.

And thus much by way of Notation: To proceed therefore to the Operations to be performed with theſe Numbers, whether reduced to ſmall Figures or not; and firſt of Addition.

Place the Numbers to be added juſt under one another, obſerving the Homogeneity of Places, as in common Numbers. Then beginning at the Right Hand, collect the Figures in the firſt Row or Column, according to their Signs, and place the Reſult under-

 neath:

neath: And so successively of all the other Columns, as in Example 1.

But if at any time this result cannot be express'd by a single Figure, it may be writ down with two or more Figures, observing the Homogeneity of Places, and then the Sum may be collected over again. But to save this trouble, it will be sufficient to reserve the Figure in mind which belongs to the next Column, and collect it with the Figures of that Column; as in Examples 2, 3.

If the Numbers to be added are reduced to small Figures, as in Example 3, their Addition will be very simple, and the Sum may also be exhibited in small Figures, by an easie Substitution of Equivalents, where there is occasion.

(1.)

$$\begin{array}{r} 2\bar{5}7\bar{3}\bar{8}42\bar{6}3 \\ 70\bar{9}\bar{8}2\bar{1}\bar{3}70 \\ 580\bar{7}30\bar{5} \\ \hline 9\bar{5}3\bar{3}\bar{6}\bar{4}21\bar{2} \end{array}$$

(2.)

$$\begin{array}{r} 64\bar{7}03\bar{9}\bar{6}\bar{8}\bar{2} \\ 4\bar{9}\bar{8}\bar{2}\bar{7}36\bar{5}1 \\ 81\bar{9}\bar{4}03\bar{7}\bar{6}5 \\ \hline 18\bar{6}\bar{4}\bar{6}\bar{4}\bar{3}\bar{8}\bar{9}4 \end{array}$$

(3.)

$$\begin{array}{lr} (m) & 2\bar{1}5\bar{3}\bar{1}40\bar{4}3\bar{1}\bar{2}\bar{1}3, \text{ \&c.} \\ (m) & 50\bar{4}\bar{2}03\bar{1}\bar{4}255\bar{1}\bar{2}, \text{ \&c.} \\ (m\text{-}1) & 4\bar{3}10\bar{2}\bar{3}10\bar{2}\bar{4}\bar{1}\bar{3}, \text{ \&c.} \\ (m\text{-}2) & 5\bar{1}3\bar{4}\bar{2}1\bar{1}032\bar{1}, \text{ \&c.} \\ (m\text{-}3) & 2\bar{1}30\bar{4}2\bar{1}03\bar{2} \text{ \&c.} \\ (m\text{-}4) & 1\bar{3}\bar{2}021\bar{2}2\bar{4}, \text{ \&c.} \\ (m\text{-}5) & 13\bar{2}\bar{2}43\bar{1}5, \text{ \&c.} \\ \hline (m+1) & 1\bar{3}33\bar{3}214\bar{1}\bar{3}\bar{4}3\bar{1}\bar{2}, \text{ \&c.} \\ \hline \end{array}$$

Sub-

Subtraction in this Arithmetick is reduced to Addition, by changing all the Signs of the Number to be ſubtracted.

Thus, if from (n) $7\bar{2}\bar{9}38\bar{4}2\bar{9}63\bar{7}$, &c. we are to ſubtract (n-2) $8\bar{1}07\bar{3}\bar{5}9\bar{2}6$, &c. the Remainder will be found as in Example 4.

(4.)

(n)	$7\bar{2}\bar{9}38\bar{4}2\bar{9}63\bar{7}$, &c.
(n-2)	$8\bar{1}07\bar{3}\bar{5}9\bar{2}6$, &c.
(n)	$7\bar{3}\bar{7}47\bar{1}5\bar{4}\bar{3}4\bar{3}$, &c.

Thus in all Caſes will Addition and Subtraction be eaſily performed: But the chief uſe of this Method will be, to eaſe the trouble of prolix Multiplications. And here, as well as in Diviſion, the firſt and moſt valuable Figures may be firſt found, and conſequently the Product may be continued to as many Places as ſhall be required, without finding any unneceſſary Figures; which is a convenience not to be had in the ordinary way of Multiplication.

Let it be propoſed to Multiply together the Numbers 8605729398715 and 389175836438, which reduced to ſmall Figures will be $1\bar{1}\bar{4}1\bar{4}\bar{3}3\bar{1}40\bar{1}\bar{3}15$ and $4\bar{1}\bar{1}2\bar{2}\bar{4}\bar{2}4\bar{4}44\bar{2}$. Write down theſe two Numbers one under the other upon a ſlip of Paper, with the Figures at equal diſtances, and then cut them aſunder. Take either of the Numbers for a Multiplier, and place it over the other in an inverted poſition, ſo as its firſt Figure

Figure may be juſt over the firſt Figure of the Multiplicand.

Moveable Multiplier

$$4\bar{1}\bar{1}2\bar{2}\bar{4}\bar{2}4\bar{4}44\bar{2}$$

Multiplicand

$$1\bar{1}\bar{4}1\bar{4}\bar{3}3\bar{1}401\bar{3}\bar{1}5$$

$$4\bar{5}\bar{6}1\bar{7}\bar{9}182549\bar{9}\bar{8}6\bar{4}50\bar{6}\bar{2}\bar{6}0\bar{6}080$$

$$\bar{1}1\bar{1}\bar{1}3\quad 112\bar{4}\quad 2\bar{6}32\quad \bar{1}\quad \bar{2}31\bar{1}$$

Product $= 4\bar{6}\bar{5}0\bar{8}\bar{6}193709\bar{9}\bar{6}0\bar{1}70\bar{7}\bar{2}\bar{6}\bar{2}\bar{3}170$

Then Multiply theſe two firſt Figures together, and their Product ($4 \times 1 = 4$) place underneath. Then move your Multiplier a place forwarder, ſo that two of its firſt Figures may be over two of the firſt Figures of the Multiplicand; and collecting their two Products ($4\times\bar{1}+\bar{1}\times 1=\bar{5}$) put their Reſult underneath in the next place. Move the Multiplier a place forwarder; and collecting the three Products ($4\times\bar{4}+\bar{1}\times\bar{1}+\bar{1}\times 1=\bar{1}\bar{6}$) put the Reſult underneath, as in the Example. Move the Multiplier, and collect the four Products ($4\times 1+\bar{1}\times\bar{4}+\bar{1}\times\bar{1}+2\times 1=11$) which write underneath as before. And ſo proceed by one ſtop at a time, as long as any Figures of the Multiplier can be over any Figures of the Multiplicand. Laſtly collect the Product into one Line, which being reduced to a common Number will be 33491419369039969 27377170.

From this Proceſs it may be obſerved, that at every new Situation of the moveable Multiplier thoſe Figures only are to be Multiply'd together, each by each, as are found over one another. And this Multipli-

tiplication is to be perform'd, and the ſeveral Products collected, according to the Rules of Specious Multiplication, wherein like Signs will make +, and unlike Signs will make — in the Product. This will always make the Products deſtroy one another, or at leaſt will depreſs and keep them low, and the Figures themſelves being always ſmall, the Reſult will be always ſmall, and often but a ſingle Figure, which is the great Compendium of this Method.

When an Approximation only is deſir'd, or when the Product is to be produced to a given Number of places, the Operation may be continued one place farther, in order to obtain ſo many Places true as are required. For ſeldom any Correction extends beyond the place immediately aforegoing, and that is generally corrected but by an unite, and very often needs no Correction at all; which will be of no ſmall convenience in the Multiplication of Decimal Fractions.

In this Method we may (if we pleaſe) begin the Proceſs of Multiplication from the loweſt places, or from the right hand, as is uſual in common Arithmetick, and then the Correction may be carry'd on continually to the next place, and ſo the Product may be always comprehended in one Line, without the uſe of any Superfluous Figures. Of this I ſhall give an inſtance in the foregoing Example.

Moveable Multiplier

$4\bar{1}\bar{1}\bar{2}\bar{2}\bar{4}2\bar{4}444\bar{2}$

Multiplicand

$1\bar{1}4{}^{1}\bar{4}\bar{3}3{}^{\bar{1}}4{}^{0}{}^{\bar{1}}\bar{3}{}^{1}5$

$3349{}^{1}41{}^{9}369039969{}^{2}7377{}^{1}70$ = *Product*

Place the moveable Multiplier inverted in such a manner, as that its last Figure $\bar{2}$ may be just over 5 the last Figure of the Multiplicand. Multiply these together ($\bar{2}\times5=\bar{1}0$) and set down the last Figure of the Product 0 just under, reserving the first Figure $\bar{1}$ for the next place. Then move the Multiplier a place forwarder, so that two of its last Figures may be over two of the last Figures of the Multiplicand, and then Multiplying and collecting you will have $\bar{1}+\bar{2}\times1+4\times5=17$. Set down 7 in the next place of the Product, and reserve 1. At the next remove you will have $1+\bar{2}\times\bar{3}+4\times1+4\times5=31$. Set down 1 and carry 3. Then $3+\bar{2}\times\bar{1}+4\times\bar{3}+4\times1+\bar{4}\times5=\bar{2}\bar{3}=\bar{3}7$. Set down 7 and carry $\bar{3}$. Then $\bar{3}+\bar{2}\times0+4\times\bar{1}+4\times\bar{3}+\bar{4}\times1+4\times5=\bar{3}=17$. Set down 7 and carry $\bar{1}$. And so proceed as long as there can be any Figures over one another, and the Product will be found as before.

This way of Multiplication is so easy, and may be made so familiar by a little Practice; that it will be but little short of *Multiplication by Inspection*, and will doubtless seem very surprizing to those who are only acquainted with the common tedious way of Multiplication: especially if we content our selves with a mental preparation of the Numbers given, or only mark those Figures that are to be changed, which by some Practice is easily attained.

The

The firſt of theſe two ways of Multiplication will be moſt convenient for interminate Numbers. As if we were to multiply (m) 30714974 1748, &c. by (n) 183609712649, &c. the Product will be found (m + n) 563956758222, &c. as may appear from the Proceſs following.

$$(n)\ 2\bar{2}4\bar{4}103̄\,13\bar{4}5\bar{1},\ \&c.$$

$$(m)\ 31\bar{3}150\bar{3}42\bar{3}5\bar{2},\ \&c.$$

$$(m+n)\ 6\bar{4}40\bar{5}7\bar{2}\bar{5}83\bar{7}\bar{8},\&c.$$

Here the Index of the firſt Figure of the Product will be m + n, or the Sum of the Indexes of the given Numbers; but it would have been m + n + 1 if there had been any increaſe from the Product of the two firſt Figures, or if there had been any correction to have been made to the Cypher, which is underſtood before the firſt Figure of the Product.

When both the Numbers to be multiply'd are interminate, as in the laſt Example, they ought to conſiſt of the ſame number of Places, or otherwiſe the greater number muſt be reduced to the leſſer, by cutting off the ſuperfluous Places: And the Product is not to be continued beyond the ſame number of Places. If but one of the number is interminate, the other muſt be reduc'd to the form of an interminate Number, either by cutting of the exceſs of places if it has more, or by ſupplying or ſuppoſing Cyphers, if it has fewer places than the interminate Number. Then the ſame reſtrictions will take place as before.

The Method of Diviſion in this Arithmetick will not be ſo ſimple or expeditious as Multiplication. After a tryal of ſeveral ways, I think this following will be the moſt commodious. Reduce the Dividend and Diviſor

to small Figures, and form a Tarifla or Table of all the Multiples of the Divisor as far as 5. Compare these Multiples with the Dividend, and with the several Remainders after the Multiples have been Subtracted, by which means you will discover the several small Figures and their Signs, to be put successively in the Quotient.

To form the Table of Multiples, set down the Divisor above, drawing a line, under which set down the Divisor over again, putting 1 over against it. Add these two together according to the Rule for Addition in small Figures, and put 2 over against their Sum. Add this last and the Divisor together, and put 3 over against their Sum. Then add this last and the Divisor together, and put 4 over against their Sum. Lastly, add this and the Divisor together, putting 5 over against their Sum. Thus will you have a Table of all the Multiples of the Divisor, as far as will be necessary.

Thus for Example, if $(m+n)\ 563956758222$, &c. $= (m+n+1)\ 1\bar{4}\bar{4}40\bar{4}\bar{3}\bar{2}\bar{4}\bar{2}222$, &c. is to be divided by $(n)\ 183609712649$, &c. $= (n)\ 2\bar{2}4\bar{4}10\bar{3}13\bar{4}5\bar{1}$, &c. the Process will be as here follows, by which the Quotient will be found $(m)\ 31\bar{3}150\bar{3}42\bar{3}5\bar{4}$, &c. $= (m)\ 307149741746$, &c.

TABLE of Multiples.

	$(n)\ 2\bar{2}4\bar{4}10\bar{3}13\bar{4}5\bar{1}$, &c.
1	$(n)\ 2\bar{2}4\bar{4}10\bar{3}13\bar{4}5\bar{1}$, &c.
2	$(n)\ 4\bar{3}\bar{3}22\bar{1}425 30\bar{2}$, &c.
3	$(n)\ 551\bar{2}3\bar{1}14\bar{2}\bar{1}5\bar{3}$, &c.
4	$(n+1)\ 1\bar{3}3444\bar{1}\bar{2}51\bar{4}0\bar{4}$, &c.
5	$(n+1)\ 1\bar{1}2\bar{2}05\bar{1}\bar{4}\bar{4}3245$, &c.

Quo-

Quotient = (*m*) $31\bar{3}150\bar{3}42\bar{3}5\bar{4}$, &c.

Dividend = $(m + n + 1)$ $\bar{1}\bar{4}440\bar{4}\bar{3}\bar{2}\bar{4}\bar{2}222$, &c.

$\bar{5}\bar{5}\bar{1}23\bar{1}1\bar{4}\bar{2}1\bar{5}3$, &c.

$1313\bar{2}\bar{4}203\bar{3}5$, &c.

$\bar{2}2\bar{4}4\bar{1}03\bar{1}\bar{3}\bar{4}5$, &c.

$\bar{1}5\bar{2}\bar{3}\bar{3}\bar{4}5\bar{1}010$, &c.

$551\bar{2}3\bar{1}14\bar{2}\bar{1}$, &c.

$3\bar{3}5\bar{1}404\bar{1}\bar{1}$, &c.

$\bar{2}2\bar{4}4\bar{1}03\bar{1}\bar{3}$, &c.

$1\bar{1}1331\bar{3}\bar{2}\bar{4}$, &c.

$\bar{1}1\bar{2}20\bar{5}144$, &c.

$\bar{1}53\bar{4}\bar{2}20$, &c.

$551\bar{2}3\bar{1}$, &c.

$1\bar{2}\bar{4}\bar{4}5\bar{1}$, &c.

$\bar{1}3\bar{3}\bar{4}\bar{4}\bar{4}$, &c.

3205, &c.

$\bar{3}33\bar{2}$, &c.

$\bar{1}533$, &c.

551, &c.

$1\bar{2}4$, &c.

$\bar{1}1\bar{2}$, &c.

$\bar{1}2$, &c.

The Index of the Quotient is found by ſubtracting the Index of the Diviſor from the Index of the Dividend, when the firſt Figure or Figures of the Diviſor are not greater than the like Figures of the Dividend. Thus $\frac{(m+n)\ 56,\ \&c.}{(n)\ 18,\ \&c.} = (m)\ 30$,&c. When they are greater, then an Unit muſt be farther Subtracted from the Index of the Dividend. Thus $\frac{(m+n+1)\ 1\bar{4}\bar{4},\ \&c.}{(n)\ 2\bar{2}4,\ \&c.} = (m)\ 31\bar{3}$, &c.

Now a little to illuſtrate this Proceſs, it may be obſerved, that the Dividend $1\bar{4}\bar{4}$, &c. or 56, &c. being compared with the ſeveral Multiples of the Diviſor in the Table, it is eaſily perceived that 55, &c. makes the neareſt approach to it, and therefore its correſpending Figure 3 muſt be made the firſt Figure of the Quotient, which I place juſt over. Under this I place its reſpective Multiple, changing the Signs and collecting, becauſe it ought to be ſubtracted. Then the remainder 131, &c. being compared with the Table, I find the neareſt to it (whether in exceſs or defect) to be $2\bar{2}$, &c. or 18, &c. belonging to 1. This therefore is made the ſecond Figure of the Quotient, and its Multiple with the Signs changed is placed under, and collected with the laſt Remainder. The Reſult, or new Remainder $\bar{1}5\bar{2}$, &c. that is — $\bar{1}52$, &c. or — 52, &c. being compared with the Table, the neareſt Multiple is 55, &c. belonging to 3, which 3 therefore is made the next Figure of the Quotient, but with the Sign — over it, becauſe this Remainder is Negative. And for this reaſon the Multiple 55, &c. is put down in its place without changing its Signs. The reſt of the Proceſs will be obvious enough, and if any ſcruple ariſes about placing the Numbers, it may eaſily be re-

moved

moved by a little attention to their reſpective Indexes.

In the Diviſion of interminate Numbers, the ſame reſtrictions are to obtain, as are already mention'd in Multiplication.

And this may ſuffice for a ſhort Summary of Negativo-affirmative Arithmetick, as to the ordinary Operations of Addition, Subtraction, Multiplication, and Diviſion. What improvements may be had from hence in the Extraction of Roots, whether of pure or affected Equations, I ſhall leave to future inquiry.

But all theſe Operations, whether in the common Decimal Arithmetick, or in this here compendiouſly deſcribed; and not only in Decimal Arithmetick, but in the ſeveral other Species, as Duodecimal, Sexageſimal, Centeſimal, *&c.* all theſe Operations (I ſay) may be much more eaſily and readily perform'd, and with equal accuracy, by an Inſtrument I have contrived call'd *Abacus*, or *the Counting-Table*, which I hope ſhortly to communicate to the inquiſitive in theſe matters, and by which all long Calculations may be very much facilitated. In the mean time this ſhort account of Negativo-affirmative Arithmetick, in Denary or Decimal Numbers, may be premis'd, by way of Introduction to the knowledge and uſe of the ſaid Inſtrument.